Deformed Fermi Surface Theory of Magneto–Acoustic Anomaly in Modulated Quantum Hall Systems Near $\nu = 1/2$

Nataliya A. Zimbovskaya* and Joseph L. Birman**

Department of Physics, The City College of CUNY, New York, NY, 10031, USA

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Abstract

We introduce a new generic model of a deformed Composite Fermion–Fermi Surface (CF–FS) for the Fractional Quantum Hall Effect near $\nu = 1/2$ in the presence of a periodic density modulation. Our model permits us to explain recent surface acoustic wave observations of anisotropic anomalies [1,2] in sound velocity and attenuation – appearance of peaks and anisotropy – which originate from contributions to the conductivity tensor due to regions of the CF–FS which are flattened by the applied modulation. The calculated magnetic field and wave vector dependence of the CF conductivity, velocity shift and attenuation agree with experiments.

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The integer and fractional quantum Hall effects (IQHE and FQHE) continue to reveal new and unexpected physics in strongly correlated 2–dimensional electron systems [3]. Recently, particular attention has been given to FQHE systems at and near half filling of the lowest Landau level (LLL). According to the Halperin, Lee and Read theory [4] at $\nu = 1/2$ each electron is decorated by two quantum flux tubes, producing a new fermionic quasiparticle, the composite fermion (CF). At $T = 0$, CFs are distributed inside the Composite Fermion–Fermi Surface (CF–FS), which is assumed to be a circle. At $\Delta\nu = 0$, the two flux tubes attached to each electron give rise via the "Chern Simons" mechanism [3] to an extra ("fictitious") magnetic field opposite to and exactly canceling the applied $\mathbf{B}$ field. When

$\nu = 1/2 \pm \Delta\nu$ ($\Delta\nu \neq 0$) the Chern–Simons field does not cancel the applied field and the CF's move in a non–zero magnetic field $\mathbf{B}_{eff}$ ($\mathbf{B}_{eff} = B - 4\pi\hbar cn/e$, where n is the electron density) which is proportional to $\Delta\nu$. In order to test the predictions of this theory it is necessary to study the motion of the carriers in this field. A sensitive tool for this purpose is surface acoustic wave (SAW) propagation which gives quantitative information about the carriers [5].

Recently [1,2] anomalous behavior was observed for the SAW velocity and attenuation near filling factor $\nu = 1/2$ when a periodic density modulation was applied. Measurements of the velocity shift $\Delta s/s$ and the attenuation Γ in the SAW response orthogonal to the modulation direction showed an unexpected effect. The minimum in $\Delta s/s$ at $\nu = 1/2$ which was observed repeatedly in non–modulated systems [5], was converted to a large maximum, when the modulation wave vector, and the magnitude of the external field which produces the modulation, were above some critical values. On further increase of the magnitude of the density modulation, the peak in the velocity shift disappeared and was again replaced by a minimum. For SAW propagation parallel to the direction of density modulation, no such anomaly was found for the response of the electron system.

The grating modulation will influence the CF system in two ways: through the direct effect of the modulating potential and through the effect of the magnetic field $\Delta B(\mathbf{r})$ proportional to the density modulation $\Delta n(\mathbf{r})$ $\left(\Delta B(\mathbf{r}) = 4\pi\hbar c\Delta n(\mathbf{r})/e\right)$. The latter was analysed recently [6] under the conditions $q << g;\ ql << 1$ ($\mathbf{q}, \mathbf{g}$ are the SAW and periodic density modulation wave vectors; l is the CF mean free path). It was shown that the corresponding component of the electron conductivity (σ_{xx}, for the SAW propagating along the x axis for $\mathbf{q} \perp \mathbf{g}$ had an additional term proportional to $(\Delta n/n)^2$ (Δn is the amplitude of the density modulation). Similar results under the same conditions were obtained in [7].

In this paper we will analyze the effect of the periodic density modulation on the CF system under conditions of the experiment [1] ($ql > 1,\ q \sim g$). The starting point of the analysis is that the periodic modulating field deforms the CF–FS analogous to the crystalline field in metals. The modulating potential wave vector $\mathbf{g}$ in this case replaces the reciprocal

lattice vector.

We will show that a modulation-induced deformation of the originally circular CF–FS can be at the origin of the observed transport anomalies. We assume that exactly at $\nu = 1/2$ the CF-FS is a circle, with radius $p_F = (4\pi n\hbar^2)^{1/2}$. In the presence of the grating modulation the CF–FS circle is distorted and can be “flattened” in the neighborhood of special points where the curvature vanishes. When $ql > 1$ such small, locally “flat” regions can under certain conditions, play a disproportionately important role in determining the magneto–conductivity response due to the unusually large density of quasiparticle states there. The response is very sensitive to local changes of the FS geometry: the flattening of the ”effective” part of the CF–FS where the CF velocity vector $\mathbf{v}$ and $\mathbf{q}$ are nearly transverse ($\mathbf{q} \cdot \mathbf{v} \approx 0$) can change the main approximation to the CF conductivity whereas the nonuniformity of the B_{eff} determines corrections to it which are small when $\Delta n/n << 1$ [6,7]. Hence the modulation–induced deformation can be the most important factor affecting the CF response functions at $ql > 1$. We introduce a concrete model which permits us to obtain analytical expressions for $\Delta s/s$ and Γ. Using appropriate parameters we obtain semiquantative agreement with experiment. The model also explains the orthogonality of response and predicts its wave-vector dependence.

Our explicit deformed CF–FS model is new, to our knowledge. A point of contact between our work and that of [7] may be their assertion of anisotropic resistivity due to the spatially averaged current and electric field in the presence of periodically modulated quasiparticle density [see eqn. (2) of ref. 7]. This assertion seems implicitly to correspond to our deformed CF–FS; the two approaches would then be equivalent when $\Delta n \ll n$. We conjecture below that the reason for the reported disappearance of peaks at the highest modulation is related to additional topological change in the CF–FS.

As a first step, assume the periodic modulation in the y-direction introduces a single Fourier component of potential V_g into a “nearly–free” particle CF model. The resulting dispersion relation is:

$$E(\mathbf{p}) = \frac{p_x^2}{2m^*} + \frac{p_y^{*2}}{2m^*} + \frac{(\hbar g)^2}{8m^*} - \sqrt{\left(\frac{\hbar g p_y^*}{2m^*}\right)^2 + V_g^2}, \qquad (1)$$

with $p_y^* = p_y - \hbar g/2$, m^* is the CF effective mass. The curvature of the 2–D CF–FS can also be directly calculated as:

$$\kappa = \left[2v_x v_y \frac{\partial v_x}{\partial p_y} - v_x^2 \frac{\partial v_y}{\partial p_y} - v_y^2 \frac{\partial v_x}{\partial p_x}\right] \Big/ v^3, \qquad (2)$$

with $v = \sqrt{v_x^2 + v_y^2}$. The curvature κ tends to zero when $p_x \to \pm p_F \sqrt{V_g/E_F}$. The importance of this is that near to these points on the CF–FS the CF velocities are nearly parallel to the y direction. When $ql \gg 1$ these parts of the CF–FS make the major contribution to the velocity shift $\Delta s/s$ and attenuation Γ of the SAW propagating in the x direction. Near these zero curvature points we will use asymptotic expressions for Eq.(1). Determining (p_{x0}, p_{y0}) by $p_{x0} = \eta p_F$, $p_{y0} = p_F\left(1 - \frac{1}{\sqrt{2}}\eta^2\right)$, where $\eta = \sqrt{V_g/E_F}$, $E_F = p_F^2/2m^*$, we can expand the variable p_y in powers of $(p_x - p_{x0})$, and keep the lowest order terms in the expansion. We obtain:

$$p_y - p_{y0} = -\eta(p_x - p_{x0}) - \frac{2}{\eta^4}\frac{(p_x - p_{x0})^3}{p_F^2}. \qquad (3)$$

Near p_{x0}, where ($|p_x - p_{x0}| < \eta^2 p_F$) the first term on the right side of Eq.(3) is small compared to the second one and can be omitted. Hence near p_{x0} we have:

$$E(\mathbf{p}) = \frac{4}{\eta^4}\frac{p_F^2}{2m^*}\left(\frac{p_x - p_{x0}}{p_F}\right)^3 + \frac{p_y^2}{2m^*}. \qquad (4)$$

The "nearly free" particle model can be used when the ratio V_g/E_F is very small. For larger V_g corresponding to $\Delta n/n$ of the order of a few percent (as in the experiment [1]) the local flattening of the CF–FS can be more significant. To analyze the contribution to the conductivity from these flattened parts we generalize Eq. (4) for $E(\mathbf{p})$ and define our model as:

$$E(\mathbf{p}) = \frac{p_0^2}{2m_1}\left|\frac{p_x}{p_0}\right|^\gamma + \frac{p_y^2}{2m_2}, \qquad (5)$$

where p_0 is a constant with the dimension of momentum, the m_i are effective masses, and γ is a dimensionless parameter which will determine the shape of the CF–FS . When $\gamma > 2$

the 2–D CF–FS looks like an ellipse flattened near the vertices $(0, \pm p_0)$. Near these points the curvature is:

$$\kappa = -\frac{\gamma(\gamma-1)}{2p_0\sqrt{m_1/m_2}}\left|\frac{p_x}{p_0}\right|^{\gamma-2} \tag{6}$$

and, $\kappa \to 0$ at $p_x \to 0$. The CF–FS will be the flatter at $(0, \pm p_0)$, the larger is the parameter γ. A separate investigation is required to establish how γ depends on modulation magnitude V_g. Here we postulated Eq. (5) as a natural generalization of Eq. (4) and we then derive the resulting SAW response.A separate investigation is required to establish how γ depends on modulation magnitude V_g. Here we postulated Eq. (5) as a natural generalization of Eq. (4) and we then derive the resulting SAW response.

In a GaAs heterostructure with a 2-D electron gas subject to a travelling SAW, piezo-electric coupling produces a longitudinal electric field which interacts with the electron gas. Taking the SAW wave vector as $(q, 0, 0)$ we obtain that the resulting velocity shift $\Delta s/s$ and SAW attenuation rate Γ are given by the following expressions [8]

$$\Delta s/s = [\alpha^2/2]\Re(1 + i\sigma_{xx}/\sigma_m)^{-1}, \tag{7}$$

$$\Gamma = -q(\alpha^2)/2\Im(1 + i\sigma_{xx}/\sigma_m)^{-1}. \tag{8}$$

In these equations, $\omega = sq$ is the SAW frequency, α is the piezoelectric coupling constant, $\sigma_m = \epsilon s/(2\pi)$ with ϵ an effective dielectric constant of the medium, σ_{xx} is the xx component of the electronic conductivity tensor; real and imaginary parts are indicated. In order to proceed we now need to establish some preliminary results. We use the semi–classical CF theory [4] in which the CF quasiparticles have charge e, and finite mass m^*. However, as described below, a particular variant of the solution of the Boltzmann equation was needed for the present work. In semiclassical CF theory the electron resistivity tensor ρ at finite $\mathbf{q}, \omega$ is the sum of a CF term and a term originating in the magnetic field of the Chern-Simons (CS) vector potential. The CS part has only off–diagonal elements,

$$(\rho^{CS})_{xy} = -(\rho^{CS})_{yx} = 4\pi\hbar/e^2. \tag{9}$$

In a strong magnetic field we have $\rho_{xy} \gg \rho_{xx}$, ρ_{yy}, and hence we can use the approximation:

$$\sigma_{xx}(\mathbf{q}) = \frac{e^4}{(4\pi\hbar)^2} \frac{\tilde{\sigma}_{xx}(\mathbf{q})}{\tilde{\sigma}_{xx}(\mathbf{q})\tilde{\sigma}_{yy}(\mathbf{q}) + \tilde{\sigma}_{xy}^2(\mathbf{q})}\,, \tag{10}$$

where $\tilde{\sigma} = (\rho^{CF})^{-1}$ is the CF conductivity.

To evaluate the CF conductivity $\tilde{\sigma}_{\alpha\beta}(\mathbf{q})$ so that we can pass smoothly to the $B_{eff} \to 0$ limit for a flattened CF–FS, we begin with the expression obtained from solution of the linearized Boltzmann equation in the presence of the magnetic field, assuming a relaxation time τ. This is:

$$\begin{aligned}\tilde{\sigma}_{\alpha\beta}(\nu) = \frac{e^2 m_c}{(2\pi\hbar)^2}\frac{1}{\Omega}\int_0^{2\pi} d\psi \Bigg\{ \exp\left[-\frac{iq}{\Omega}\int_0^{\psi} V_x(\psi'')d\psi''\right] v_\alpha(\psi)\, \times \\ \times \int_{-\infty}^{\psi} \exp\left[\frac{iq}{\Omega}\int_0^{\psi'} v_x(\psi')d\psi' + \frac{1}{\Omega\tau}(\psi'-\psi)\right] v_\beta(\psi')d\psi' \Bigg\}.\end{aligned} \tag{11}$$

Here $V_{\alpha,\beta}$ are the CF velocity components $(\alpha, \beta = x, y)$; $\Omega = |e|B_{eff}/m_c c$ is their cyclotron frequency; ψ is the angular coordinate on the CF cyclotron orbit, ($\psi = \Omega\theta$; θ is the time of the CF motion along the cyclotron orbit). We have taken $\omega\tau \ll 1$. We proceed [9] as follows. Express the velocity components $v_\beta(\psi')$ as Fourier series:

$$v_\beta(\psi') = \sum_k v_{k\beta} \exp(ik\psi'). \tag{12}$$

Introducing a new variable η:

$$\eta \equiv \left(\frac{1}{\tau} + ik\Omega + iqv_x(\psi)\right)\theta + iq\int_0^{\theta}[v_x(\psi + \Omega\theta') - v_x(\psi)]d\theta'; \qquad \theta = (\psi' - \psi)/\Omega \tag{13}$$

and substituting (12) and (13) into (11) we obtain:

$$\tilde{\sigma}_{\alpha\beta}(\nu) = \frac{e^2 m_c \tau}{(2\pi\hbar)^2}\sum_k v_{k\beta}\int_{-\infty}^{0} e^{\eta} d\eta \int_0^{2\pi} \frac{v_\alpha(\psi)\exp(ik\psi)d\psi}{1 + ik\Omega\tau + iqv_x(\psi + \tilde{\theta}(\eta)\Omega)\tau}\,. \tag{14}$$

To proceed we can transform the integral over ψ in (14) to an integral over the CF–FS. Reexpressing the element of integration as $m_c d\psi = d\lambda/|v|$ ($d\lambda$ is the element of length along the Fermi Arc), and replacing m_c by a suitable combination of m_1, m_2 of our model

(5); e.g. for an ellipse $m_c = \sqrt{m_1 m_2}$, we can now parameterize the dispersion equation of our model (5) as follows:

$$p_x = \pm p_0 |\cos t|^{2/\gamma}; \qquad\qquad p_y = p_0 \sqrt{m_2/m_1}\,\sin t, \tag{15}$$

where $0 \leq t \leq 2\pi$, and the $+$ and $-$ signs are chosen corresponding to normal domains of positive and negative values of the cosine. Where $ql \gg 1$, the leading term in the resulting formula originates from parts of the CF–FS where $v_x \approx 0$. Expanding it in powers of $(ql)^{-1}$ and keeping the main term in the expansion we obtain:

$$\tilde{\sigma}_{yy}(\nu) = \frac{b}{2}\frac{e^2 p_0}{4\pi\hbar^2}\frac{l}{(ql)^\mu}(S_{+\mu}(\Omega\tau) + S_{-\mu}(\Omega\tau)) \tag{16}$$

where: $S_{\pm\mu}(\Omega\tau) = \int\limits_{-\infty}^{0} e^\eta (1 \mp i\Omega\tau(1 \pm \eta\delta_0))^{\mu-1} d\eta$ and δ_0 is a small dimensionless constant of the order of $\omega\tau$. Here for convenience we introduced $\mu = 1/(\gamma - 1)$ which is a dimensionless parameter ($0 \neq \mu \leq 1$), with $\mu = 1$, or $\gamma = 2$ corresponding to the case that the CF–FS is an ellipse. In these variables, the CF mean-free-path ℓ is equal to:

$$\ell = \frac{\mu+1}{2\mu}\frac{p_0\tau}{m_1}. \tag{17}$$

Passing to the limit $B_{eff} = 0$ we have:

$$\tilde{\sigma}_{yy}\left(\nu = \frac{1}{2}\right) = \frac{be^2 p_0}{4\pi\hbar^2}\frac{\ell}{(q\ell)^\mu}\,. \tag{18}$$

In this equation $b = 4\mu^2/(\mu+1)\sqrt{m_1/m_2}[\sin(\pi\mu/2)]^{-1}$. This expression eqn.(18) predicts that measuring the q-dependence of the conductivity exactly at $B_{eff} = 0$ ($\nu = 1/2$) can give the deformation parameter μ. When the CF–FS is an undeformed circle ($m_1 = m_2 = m^*$) then $b = 2$ and the result is identical to the corresponding result obtained in [4]. It is worth emphasizing that when the flattening of the CF-FS is strong, with $\gamma \gg 1$, the quantity $\mu \approx 0$ and the CF conductivity will be enhanced compared to the circular case, and it will be effectively independent of q (See eqn.(16)). Independence of q has been found experimentally [1]. For small $\Omega\tau$, ($\Omega\tau\omega\tau < 1$) one can expand the functions $S_{\pm\mu}(\Omega\tau)$ ($\mu \neq 1$) in powers of $\delta_0\Omega\tau$:

$$S_{\pm\mu}(\Omega\tau) = (1 \mp \Omega\tau)^{\mu-1}\left[1 + \sum_{r=1}^{\infty} \frac{(1-\mu)(2-\mu)...(r-\mu)}{(1 \mp i\Omega\tau)^r}(i\delta_0\Omega\tau)^r\right]. \tag{19}$$

Keeping the terms larger than $(\Omega\tau)^3$ one has:

$$\tilde{\sigma}_{yy} = \tilde{\sigma}_{yy}\left(\nu = \frac{1}{2}\right)[1 - a^2(\Omega\tau)^2 + i\xi\Omega\tau]. \tag{20}$$

Here $a^2 = ((1-\mu)(2-\mu)/2)(1+2\delta_0^2)$ and $\xi = (1-\mu)\delta_0$ are positive constants. For sufficiently small values of the parameter μ (significant flattening of the effective parts of the CF–FS) the constant a^2 is of the order of unity and the constant ξ is small compared to unity, because of the small factor δ_0. Other components of the CF conductivity tensor can be calculated similarly.

Substituting the results into (10), we can obtain the expression for the electron conductivity component σ_{xx}. Then using (7),(8) we have:

$$\frac{\Delta s}{s} = \frac{\alpha^2}{2}\frac{1 + \xi\Omega\tau\bar{\sigma}}{1 + \bar{\sigma}^2}\left(1 - \frac{2\xi\Omega\tau\bar{\sigma}}{1 + \bar{\sigma}^2} - \frac{\bar{\sigma}^2}{1 + \bar{\sigma}^2}(2a^2 - \xi^2)(\Omega\tau)^2\right); \tag{21}$$

$$\Gamma = q\frac{\alpha^2}{2}\frac{\bar{\sigma}^2}{1 + \bar{\sigma}^2}\left(1 - \frac{2\xi\Omega\tau\bar{\sigma}}{1 + \bar{\sigma}^2} - \frac{a^2\bar{\sigma}^2}{1 + \bar{\sigma}^2}(\Omega\tau)^2\right). \tag{22}$$

Here $\bar{\sigma} = \sigma_{xx}(\nu = 1/2)/\sigma_m$. Expression (21) and (22) are the new results of our theory. They predict peaks both in the SAW attenuation and velocity shift at $\nu = 1/2$; the peaks arise due to distortion of the CF–FS in the presence of the density modulation. When the CF–FS flattening is strong ($\mu \ll 1$) the magnitude of the peak of the velocity shift is practically independent of the SAW wave vector q. Also these anomalies are not sensitive to any relation between q and the density modulation wave vector g. As was observed repeatedly [1,2] the peaks appear when the magnitude of the modulating potential and its wave vector are sufficiently large. These quantities V_g and g determine the character and amount of distortion of the CF–FS, by changing γ in our model.

We now suggest an explanation for the observed disappearance of the SAW peak in $\Delta s/s$ when the magnitude of density modulation was at highest measured values. In metals it is known [10,11] that external factors, as well as changes in electron density can cause

changes in FS topology such as in the connectivity. These changes are sensitively reflected in the response functions. We suggest this can occur in the CF case. A topological change of the CF–FS connectivity can be caused by increased magnitude of modulating field and correspondingly increased quasiparticle density modulation amplitude Δn. Changing the CF–FS connectivity can lead to the disappearance of the flattening of the effective parts of the CF–FS. In this case the anomalous maximum in the magnetic field dependence of $\Delta s/s$ will be replaced by minimum. Thus assuming the relevance of such a CF topological transition, we can explain the disappearance of the peak in the SAW velocity shift under increase of the modulation strength. Additional experimental consequences of our model and more details of the theory including the analysis of the contributions to the 2DEG respose arising due to the additional nonuniform magnetic field $\Delta B(\mathbf{r})$ will be presented elsewhere [12].

We again remark that our work is based on the charged CF picture for FQHE, for example as derived at $\nu = 1/2$ in ref. [4]. We followed previous work by assuming the CF–FS exists, as supported also by a theoretical study [13]. The relevant magnetic symmetry translation [14] which would replace Bloch wave vector $\mathbf{k}$ by a "good" quantum index for state labels and transport theory, have not been used to our knowledge. An alternate picture for the FQHE also derived from a Chern–Simons approach, gives the quasiparticles at $\nu = 1/2$ as neutral dipolar objects, with the Hall current being carried by a set of collective magneto–plasmon oscillators. To our knowledge, a magneto–transport theory based on this second picture does not exist at present, so we are not able to compare our results with any derived from that picture.

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email is: *nzimbov@scisun.sci.ccny.cuny.edu and **birman@scisun.sci.ccny.cuny.edu

1. R.L.Willet, K.W.West and L.N.Pfeiffer, Phys. Rev. Lett., **78**, 4478 (1997).

2. J.H.Smet, K. von Klitzing, D.Weiss and W.Wegscheider, Phys. Rev. Lett., **80**, 4538 (1998).

3. "The Quantum Hall Effect" ed R.E.Prange and S.M.Girvin (Springer, NY 1987); "Perspectives in Quantum Hall Effect" ed. S. das Sarma and A.Pinczuk (J.Wiley, 1997).

4. B.I.Halperin, P.A.Lee and N.Read, Phys. Rev. B **47**, 7312 (1993); S.N.Simon and B.I.Halperin, Phys. Rev. B **48**, 17368 (1993); A.Stern and B.I.Halperin, Phys. Rev. B **52**, 5840 (1995).

5. R.L.Willet. Advances of Physics, **46**, 447, (1997).

6. A.D.Mirlin, P.Wolfle, Y.Levinson and O.Entin-Wohlman, cond-mat/9802140.

7. Felix v. Oppen, Ady Stern and B.I.Halperin, Phys. Rev. Lett., **80**, 4494 (1998).

8. K.A.Inbergrigsten, J. Appl. Phys. **40**, 2681 (1969); P.Bierbaum, Appl. Phys. Lett. **21**, 595 (1972).

9. N.A.Zimbovskaya, V.I.Okulov, A.Yu.Romanov and V.P.Silin, Fiz. Met. Metalloved., **62**, 1095 (1986) (in Russian); N.A.Zimbovskaya, Fiz. Nizk. Temp. **20**, 441 (1994); [Sov. Low Temperature Physics, **20**, 324 (1994)]. N.A.Zimbovskaya, "Local Geometry of the Fermi Surface and High Frequency Phenomena in Metals." "Nauka", Ekaterinburg, 1996 (in Russian).

10. I.M.Lifshitz, Zh. Eksp. Teor. Fiz. **38**, 1569 (1960) [Sov. Phys JETP **11**, 1130 (1960)].

11. Ya.M.Blanter, M.I.Kaganov, A.V.Pantsulava and A.A.Varlamov, Physics Reports, **245**, 159 (1994).

12. N.A.Zimbovskaya and J.L.Birman (in preparation).

13. P.Kopietz and G.E.Castilla, Phys. Rev. Lett. **78**, 314 (1997).

14. P.B.Wiegmann and A.Zabrodin, Nuclear Physics B, **422**, 495 (1994).